Module 4: RF Network Analysis and S-Parameters

This module is designed to provide an exhaustive and numerically-driven understanding of RF network analysis, with a particular focus on Scattering Parameters (S-parameters). We will first thoroughly explore why traditional circuit analysis methods fall short in the high-frequency domain. Then, we will dive into the definition, physical meaning, and conceptual measurement of S-parameters, followed by detailed numerical examples illustrating their calculation and interpretation. Finally, we will apply S-parameters to analyze complex RF circuits, including cascaded networks, and perform critical stability analysis with step-by-step numerical demonstrations.

4.1 Limitations of Z, Y, H, ABCD Parameters at RF

At lower frequencies, circuit analysis commonly employs parameters like Impedance (Z-parameters), Admittance (Y-parameters), Hybrid (H-parameters), and ABCD parameters. These methods fundamentally describe a multi-port network by relating the total voltages and currents at its various terminals. For instance, Z-parameters relate port voltages to port currents (V = Z * I), while Y-parameters relate port currents to port voltages (I = Y * V).

However, as we transition into the Radio Frequency (RF) and microwave frequency ranges (typically above a few tens or hundreds of MHz), the underlying assumptions for these traditional parameters break down, leading to significant practical and theoretical limitations:

1. **Impossibility of Ideal Open-Circuit and Short-Circuit Measurements:**
    - **The Problem: All Z, Y, H, and ABCD parameters are mathematically defined under specific terminal conditions that are extremely difficult, if not impossible, to achieve ideally at RF.**
        - **To find Z-parameters, you must apply a current to one port and measure voltages at all ports while keeping all other ports open-circuited (meaning zero current flows into or out of them).**
        - **To find Y-parameters, you must apply a voltage to one port and measure currents at all ports while keeping all other ports short-circuited (meaning zero voltage across them).**
    - **Real-World Deviations at RF:**
        - **"Open Circuit" Illusion: At RF, simply leaving a port unconnected does not create a true open circuit. The physical structure of the unconnected terminal (e.g., the end of a printed circuit board trace, a component lead) will inherently have parasitic capacitance to ground or to other**

nearby conductors. This parasitic capacitance provides a path for current at high frequencies, meaning the current is not zero, violating the open-circuit condition. The higher the frequency, the lower the impedance of this parasitic capacitance, making the "open" even less ideal.
    - "Short Circuit" Illusion: Similarly, connecting a "short" wire across a port to achieve a zero-voltage short circuit is also problematic. Any physical wire, no matter how short, possesses inherent parasitic series inductance. At high frequencies, this parasitic inductance ($XL=2*pi*f*L$) creates a non-zero impedance, meaning the voltage across the "shorted" terminals is not zero. The higher the frequency, the higher the impedance of this parasitic inductance, making the "short" even less ideal.
  - Practical Consequence: Due to these unavoidable parasitic effects, measurements performed under "open" or "short" conditions at RF are inaccurate and do not represent the true intrinsic parameters of the device. The measured values would include the unknown and frequency-dependent effects of the measurement setup itself.
2. Stability Issues with Active Devices under Extreme Terminations:
  - Active Device Vulnerability: Many RF circuits contain active devices, such as transistors (e.g., MOSFETs, BJTs). These devices are designed to amplify signals, meaning they possess inherent gain. However, this gain can lead to instability if positive feedback occurs.
  - Oscillation Trigger: When an active device is subjected to extreme load conditions like perfect open circuits (infinite impedance) or perfect short circuits (zero impedance) at its input or output ports (which are required for Z, Y, H, ABCD measurements), it can become unstable. This instability often manifests as oscillation, where the device spontaneously generates its own unwanted signals, converting DC power into RF power.
  - Measurement Breakdown: An oscillating device cannot be accurately characterized. Its behavior is no longer predictable or linear. Attempting to measure its Z, Y, H, or ABCD parameters under these unstable conditions would yield meaningless or rapidly fluctuating results. This is a critical limitation because active devices are the core of most RF communication systems (amplifiers, mixers, oscillators).
3. Neglect of Wave Propagation Effects (Distributed Nature):
  - Lumped Element Assumption: Traditional Z, Y, H, ABCD parameters are rooted in lumped element circuit theory. This theory assumes that the dimensions of components and

interconnections are negligible compared to the signal wavelength. Under this assumption, voltages and currents are considered uniform across a component, and signal changes occur instantaneously throughout the circuit.

- Distributed Behavior at RF: As established in Module 1, at RF, the wavelength of the signal can be comparable to, or even smaller than, the physical dimensions of the circuit's interconnections (e.g., traces on a printed circuit board, coaxial cables). In this regime, the signal propagates as a wave. Voltage and current are no longer constant along a wire; instead, they vary significantly in magnitude and phase. This leads to phenomena like reflections (when waves encounter impedance mismatches) and standing waves.
- Inability to Differentiate Waves: Z, Y, H, ABCD parameters describe total voltages and currents. They do not intrinsically differentiate between the portion of a signal wave that is traveling forward (incident wave) and the portion that is traveling backward (reflected wave). This distinction is fundamental to understanding power flow, reflections, and impedance matching in RF systems. Without this differentiation, a complete picture of high-frequency circuit behavior is impossible.

Because of these profound limitations, RF engineers primarily rely on Scattering Parameters (S-parameters). S-parameters elegantly bypass these issues by focusing on incident and reflected power waves under well-behaved, matched termination conditions.

4.2 S-Parameters (Scattering Parameters)

S-parameters provide a powerful and practical framework for analyzing, designing, and characterizing RF and microwave networks. They describe the behavior of a network by relating the incident and reflected power waves at its ports.

Definition and physical significance of S-parameters:

Instead of total voltages and currents, S-parameters work with normalized incident waves (an) and reflected waves (bn) at each port 'n' of a network. These waves are defined such that their squared magnitudes represent power:

- $|a_n|^2$ represents the power incident on port 'n'.
- $|b_n|^2$ represents the power reflected from port 'n'.

These waves are normalized with respect to a specific reference impedance, typically the standard characteristic impedance of RF systems, which is 50 Ohms.

For any N-port network, the relationship between the reflected waves and incident waves is expressed by the S-matrix equation:

$[b]=[S]*[a]$

Where:

- **$[b]$ is a column vector of reflected waves ($b_1,b_2,...,b_N$).**
- **$[a]$ is a column vector of incident waves ($a_1,a_2,...,a_N$).**
- **$[S]$ is the N x N S-parameter matrix.**

Let's focus on the most common scenario in RF: a Two-Port Network. This represents a vast majority of RF components like amplifiers, filters, attenuators, mixers, and so on, which have a defined input (Port 1) and output (Port 2).

For a two-port network, the relationships are explicitly written as:

$b_1=S_{11}*a_1+S_{12}*a_2$

$b_2=S_{21}*a_1+S_{22}*a_2$

Each S-parameter, S_{ij}, is a complex number (possessing both magnitude and phase) and is defined as the ratio of a reflected wave (b_i) to an incident wave (a_j), under the crucial condition that all other ports are terminated with the characteristic impedance (Z_0). Terminating a port with Z_0 implies that there are no reflections from that termination, effectively making the incident wave at that port zero ($a_k=0$ for $k=j$).

Let's delve into the physical significance of each of the four S-parameters for a two-port network:

- **S_{11} (Input Reflection Coefficient):**
  - **Definition: $S_{11}=b_1/a_1$, when $a_2=0$. (This means Port 2 is terminated with a perfect 50 Ohm load, so no signal is incident on Port 2 from the outside.)**
  - **Physical Significance: S_{11} quantifies how well the input port (Port 1) of the device is matched to the system's characteristic impedance (e.g., 50 Ohms). It represents the fraction of the incident power wave at the input that is reflected back from the input port. A larger magnitude of S_{11} means more reflection and thus a poorer match.**
  - **Interpretation:**

        - **If $|S11|=0$: Perfect input match. All incident power enters the device; none is reflected. This is ideal.**
        - **If $|S11|=1$: Complete reflection (total mismatch). All incident power is reflected back. This indicates an open circuit, a short circuit, or a highly reactive termination.**
        - **Typically, for a good match, you aim for $|S11|$ to be a small value (e.g., less than 0.1).**
    - **Relation to Input Return Loss (RL_in): This is often expressed in decibels (dB).**
      $RLin=-20*\log10(|S11|)$ dB.
      A higher positive value (e.g., 20 dB, 30 dB) for Return Loss indicates a better match (less reflection). For example, if $|S11|=0.1$, $RLin=-20*\log10(0.1)=-20*(-1)=20$ dB. If $|S11|=0.5$, $RLin=-20*\log10(0.5)=-20*(-0.301)\approx6.02$ dB. A lower return loss value implies more power reflected.
- **S21 (Forward Transmission Coefficient / Forward Gain):**
    - **Definition: $S21=b2/a1$, when $a2=0$. (Port 2 terminated in 50 Ohms.)**
    - **Physical Significance: S21 describes the transmission of a signal from the input port (Port 1) to the output port (Port 2). It represents the ratio of the transmitted wave emerging from Port 2 to the incident wave entering Port 1. It is the most important parameter for characterizing the gain or loss of an RF circuit.**
    - **Interpretation:**
        - **For an amplifier, $|S21|>1$: The network provides gain. The power gain is equal to $|S21|2$.**
        - **For a passive device (like a filter or attenuator), $|S21|\leq1$: The network either passes the signal with some loss (attenuation) or transmits it perfectly (no loss).**
    - **Expressed in dB: Gain or Insertion Loss is expressed as $20*\log10(|S21|)$ dB. For gain, the dB value is positive; for loss (attenuation), it's negative. For instance, if $|S21|=10$, Gain = $20*\log10(10)=20$ dB. If $|S21|=0.5$, Loss = $20*\log10(0.5)\approx-6.02$ dB.**
- **S12 (Reverse Transmission Coefficient / Reverse Isolation):**
    - **Definition: $S12=b1/a2$, when $a1=0$. (Port 1 terminated in 50 Ohms.)**
    - **Physical Significance: S12 quantifies the transmission of a signal from the output port (Port 2) back to the input port (Port 1). It is a measure of the reverse isolation or reverse gain of the device.**
    - **Interpretation:**
        - **For an ideal amplifier, $|S12|$ should be very small (close to 0). This indicates excellent reverse isolation, meaning signals originating from the output (e.g., reflections from a mismatched load, or other signals at the output) do not significantly feed back to or interfere with the input signal. High reverse isolation is crucial for amplifier stability.**

- If $S12=0$, the device is considered unilateral. This is an ideal condition rarely achieved perfectly in practice, but often strived for in amplifier design.
    - Expressed in dB: Reverse Isolation is typically given as $-20*\log10(|S12|)$ dB. A higher positive value (e.g., 30 dB, 40 dB) signifies better isolation. For example, if $|S12|=0.01$, Isolation = $-20*\log10(0.01)=-20*(-2)=40$ dB.
- S22 (Output Reflection Coefficient):
    - Definition: $S22=b2/a2$, when $a1=0$. (Port 1 terminated in 50 Ohms.)
    - Physical Significance: S22 quantifies how well the output port (Port 2) of the device is matched to the system's characteristic impedance. It represents the fraction of the incident power wave at the output that is reflected back from the output port.
    - Interpretation: Similar to S11, a smaller magnitude of S22 indicates a better output match.
    - Relation to Output Return Loss (RL_out): $RLout=-20*\log10(|S22|)$ dB.

Numerical Example 4.2.1: Interpreting S-Parameters (Detailed)

Imagine a single-stage RF amplifier designed for operation at 1.8 GHz. After fabricating and testing it with a Vector Network Analyzer (VNA) at 1.8 GHz with 50 Ohm terminations, the following S-parameters are measured:

$S11=0.15\angle 135^\circ$ (Magnitude and phase)

$S21=4.5\angle 30^\circ$

$S12=0.02\angle -15^\circ$

$S22=0.25\angle -70^\circ$

Let's interpret each of these values:

1. Input Reflection Coefficient (S11):
    - Magnitude: $|S11|=0.15$
    - Phase: 135°
    - Interpretation: A magnitude of 0.15 means that 15% of the input voltage wave (or 2.25% of the input power, since Power is proportional to Voltage squared, so $0.15^2=0.0225$) is reflected back from the amplifier's input.
    - Return Loss Calculation:
      $RLin=-20*\log10(|S11|)=-20*\log10(0.15)$
      $\log10(0.15)\approx-0.8239$
      $RLin=-20*(-0.8239)\approx16.48$ dB

- **Conclusion: A return loss of approximately 16.48 dB is generally considered a reasonably good input match for many RF applications, implying that most of the incident power is accepted by the amplifier.**

2. **Forward Transmission Coefficient (S21):**
   - **Magnitude: $|S21|=4.5$**
   - **Phase: 30°**
   - **Interpretation: A magnitude of 4.5 means that the voltage wave at the output is 4.5 times larger than the incident voltage wave at the input (assuming 50 Ohm terminations at both ports).**
   - **Power Gain Calculation:**
     Power Gain (linear) $=|S21|^2=(4.5)^2=20.25$
     This means the output power is 20.25 times the input power.
   - **Gain in dB:**
     Gain (dB) $=20*\log10(|S21|)=20*\log10(4.5)$
     $\log10(4.5)\approx0.6532$
     Gain (dB) $=20*(0.6532)\approx13.06$ dB
   - **Conclusion: The amplifier provides a healthy gain of approximately 13.06 dB, which is typical for a single-stage RF amplifier.**
3. **Reverse Transmission Coefficient (S12):**
   - **Magnitude: $|S12|=0.02$**
   - **Phase: -15°**
   - **Interpretation: A magnitude of 0.02 indicates that only 2% of a signal incident on the output port would be transmitted back to the input port. This signifies good isolation.**
   - **Reverse Isolation Calculation:**
     Isolation (dB) $=-20*\log10(|S12|)=-20*\log10(0.02)$
     $\log10(0.02)\approx-1.699$
     Isolation (dB) $=-20*(-1.699)\approx33.98$ dB
   - **Conclusion: An isolation of nearly 34 dB is excellent. This implies that the amplifier is very close to being unilateral, which helps in preventing unwanted feedback and ensures stable operation.**
4. **Output Reflection Coefficient (S22):**
   - **Magnitude: $|S22|=0.25$**
   - **Phase: -70°**
   - **Interpretation: A magnitude of 0.25 means that 25% of the incident voltage wave at the output port (if there were an incident wave, e.g., from a load mismatch) would be reflected back.**
   - **Return Loss Calculation:**
     $RLout=-20*\log10(|S22|)=-20*\log10(0.25)$
     $\log10(0.25)\approx-0.6021$
     $RLout=-20*(-0.6021)\approx12.04$ dB

- **Conclusion: An output return loss of 12.04 dB indicates a moderately good output match. It's not as good as the input match, suggesting that a small amount of power might be reflected from the output back into the amplifier if the load connected to it is not perfectly 50 Ohms.**

Measurement of S-parameters using Vector Network Analyzer (VNA) - conceptual:

The Vector Network Analyzer (VNA) is the workhorse instrument for measuring S-parameters. It's called "Vector" because it measures both the magnitude and phase of the S-parameters.

Conceptual Working Principle (Step-by-Step with Analogy):

Imagine you have a complex black box (your Device Under Test, or DUT) with two access points (ports). You want to understand how signals behave when they interact with this box.

1. **Signal Generation (The "Test Beam"):**
    - **The VNA starts by generating a very precise, stable, and tunable RF signal, much like shining a light beam. This is your "incident wave" (a1 or a2).**
    - **Example: The VNA might generate a 1 GHz sine wave.**
2. **Launching the Test Beam and Monitoring its Path:**
    - **The VNA directs this test signal into one of the DUT's ports (let's say Port 1).**
    - **Before the signal even reaches the DUT, the VNA has internal components called directional couplers. Think of a directional coupler as a "traffic cop" that can accurately distinguish between signals traveling in one direction (towards the DUT) and signals traveling in the opposite direction (reflected from the DUT).**
    - **The VNA uses these couplers to precisely measure the strength and phase of the outgoing incident wave (a1).**
3. **Measuring the Response (Reflected and Transmitted Beams):**
    - **As the incident wave (a1) enters the DUT at Port 1:**
        - **Some of it might be reflected back from Port 1 (this is the b1 wave). The VNA measures this reflected wave using the same directional coupler at Port 1.**
        - **Some of it might be transmitted through the DUT and emerge from Port 2 (this is the b2 wave). To accurately measure this, Port 2 of the DUT needs to be terminated with a perfect, reflection-less load (usually 50 Ohms). This ensures that any wave emerging from Port 2 is truly**

transmitted and doesn't get reflected back into the DUT from the outside. The VNA measures this transmitted wave (b2) using a directional coupler at Port 2.
    - **Example: If you send a 1V, 0-degree phase wave into Port 1:**
        - **You might measure 0.15V, 135-degree phase reflected from Port 1 (b1).**
        - **You might measure 4.5V, 30-degree phase transmitted out of Port 2 (b2).**

4. **Calculating the First Set of S-parameters (S11 and S21):**
    - **From these measurements, the VNA calculates:**
        - **$S11=b1/a1=(0.15\angle 135^\circ)/(1\angle 0^\circ)=0.15\angle 135^\circ$**
        - **$S21=b2/a1=(4.5\angle 30^\circ)/(1\angle 0^\circ)=4.5\angle 30^\circ$**
5. **Reversing the Flow (for S12 and S22):**
    - **To get the remaining S-parameters (S12 and S22), the VNA reverses the process. It now sends the test signal (a2) into Port 2 of the DUT.**
    - **Crucially, Port 1 is now terminated with a perfect 50 Ohm load to prevent external reflections from interfering with the measurement.**
    - **The VNA measures:**
        - **The reflected wave (b2) from Port 2.**
        - **The transmitted wave (b1) that emerges from Port 1 (this is the reverse transmission).**
6. **Calculating the Second Set of S-parameters (S12 and S22):**
    - **From these new measurements, the VNA calculates:**
        - **S22=b2/a2**
        - **S12=b1/a2**
7. **Frequency Sweep:**
    - **The entire sequence (steps 1-6) is rapidly repeated across a wide range of frequencies. The VNA "sweeps" through hundreds or thousands of frequency points to characterize the DUT's S-parameters over its entire operating bandwidth.**
    - **Example: For an amplifier designed for 1.7 GHz to 2.0 GHz, the VNA would sweep from 1.6 GHz to 2.1 GHz, taking measurements at 1000 points.**
8. **Display and Analysis:**
    - **The VNA's sophisticated software processes these complex number measurements and displays them in various useful formats:**
        - **Rectangular Plots: Magnitude (in dB) vs. Frequency, Phase (in degrees) vs. Frequency for each S-parameter. This is great for seeing gain, loss, and matching across a band.**

- Smith Charts: Reflection coefficients (S11, S22) are often plotted on a Smith Chart, which graphically shows impedance matching and stability regions.

Crucial Point: Calibration: Before any actual device measurement, the VNA must undergo a precise calibration procedure. This involves connecting known standards (e.g., a perfect open circuit, a perfect short circuit, a perfect 50 Ohm load, and a direct connection "thru" between VNA ports) to the measurement cables. The VNA uses these known responses to mathematically remove the parasitic effects of the test cables and adapters, ensuring that the S-parameters measured truly represent only the DUT. Without proper calibration, measurements would be meaningless.

4.3 Relationship between S-parameters and other parameters

While S-parameters are the preferred language in RF, occasionally it becomes necessary to convert between S-parameters and other network parameters (Z, Y, H, ABCD). This typically arises when integrating RF components into a larger system simulated or designed using different parameter sets, or when comparing with older datasheets.

The conversion process is based on the fundamental definitions of the normalized incident and reflected waves (an and bn) in terms of the total port voltages (Vn) and currents (In), relative to the system's characteristic impedance (Z0).

The relationships are:

Vn=Z00.5∗(an+bn)

In=Z0−0.5∗(an−bn)

Conversely:

an=(Vn+Z0∗In)/(2∗Z00.5)

bn=(Vn−Z0∗In)/(2∗Z00.5)

Using these fundamental expressions, complex algebraic manipulations allow us to derive conversion formulas. These formulas are usually for a 2-port network. Manual calculation is extremely tedious due to the involvement of complex numbers, so in practice, these conversions are almost exclusively performed by specialized RF design software (like Keysight ADS, Cadence Virtuoso, Ansys HFSS, etc.).

Let's look at one example of such a conversion formula, S-parameters to Z-parameters, for a 2-port network, operating with characteristic impedance Z0:

First, we need to calculate a determinant-like term, often denoted as ΔS:

$\Delta S = S11 * S22 - S12 * S21$

Then, the Z-parameters can be found using the following equations:

$Z11 = Z0 * ((1+S11)(1-S22)+S12S21)/((1-S11)(1-S22)-S12S21)$

$Z12 = Z0 * (2 * S12)/((1-S11)(1-S22)-S12S21)$

$Z21 = Z0 * (2 * S21)/((1-S11)(1-S22)-S12S21)$

$Z22 = Z0 * ((1-S11)(1+S22)+S12S21)/((1-S11)(1-S22)-S12S21)$

Notice that the denominator for all Z-parameters is the same term: $(1-S11)(1-S22)-S12S21$, which is often written as $(1-S11)(1-S22)-S12S21 = DS$ or $(1-S11)(1-S22)-\Delta S$.

Similarly, here are the formulas for S-parameters to Y-parameters:

Again, calculate a common denominator term, denoted as DY:

$DY = (1+S11)(1+S22)-S12S21$

Then, the Y-parameters are:

$Y11 = (1/Z0) * ((1-S11)(1+S22)+S12S21)/DY$

$Y12 = (1/Z0) * (-2 * S12)/DY$

$Y21 = (1/Z0) * (-2 * S21)/DY$

$Y22 = (1/Z0) * ((1+S11)(1-S22)+S12S21)/DY$

Numerical Example 4.3.1: Converting S-parameters to Z-parameters (Conceptual Walkthrough)

Let's use the amplifier S-parameters from Example 4.2.1 at 1.8 GHz, with $Z0 = 50$ Ohms:

$S11 = 0.15 \angle 135^\circ$

$S21 = 4.5 \angle 30^\circ$

$S12 = 0.02 \angle -15^\circ$

$S22=0.25\angle -70^\circ$

Step 1: Convert all S-parameters to rectangular form.

This is essential for complex number multiplication and subtraction.

- **$S11=0.15*(\cos 135^\circ + j\sin 135^\circ)=0.15*(-0.7071+j0.7071)=-0.106065+j0.106065$**
- **$S12=0.02*(\cos(-15^\circ)+j\sin(-15^\circ))=0.02*(0.9659-j0.2588)=0.01932-j0.005176$**
- **$S21=4.5*(\cos 30^\circ + j\sin 30^\circ)=4.5*(0.8660+j0.5)=3.897+j2.25$**
- **$S22=0.25*(\cos(-70^\circ)+j\sin(-70^\circ))=0.25*(0.3420-j0.9397)=0.0855-j0.234925$**

Step 2: Calculate $\Delta S=S11S22-S12S21$

This involves multiplying two pairs of complex numbers and then subtracting the results.

- **$S11S22=(-0.106065+j0.106065)*(0.0855-j0.234925)$**
  $=(-0.106065)(0.0855)+(-0.106065)(-j0.234925)+(j0.106065)(0.0855)+(j0.106065)(-j0.234925)$
  $=-0.00907+j0.0249+j0.00907+0.0249$ (after $j2=-1$)
  $=0.01583+j0.03397$
- **$S12S21=(0.01932-j0.005176)*(3.897+j2.25)$**
  $=(0.01932)(3.897)+(0.01932)(j2.25)+(-j0.005176)(3.897)+(-j0.005176)(j2.25)$
  $=0.07529+j0.04347-j0.02016+0.011646$ (after $j2=-1$)
  $=0.086936+j0.02331$
- **$\Delta S=(0.01583+j0.03397)-(0.086936+j0.02331)$**
  $=(0.01583-0.086936)+j(0.03397-0.02331)$
  $=-0.071106+j0.01066$

Step 3: Calculate the common denominator term for the Z-parameter formulas.

Denominator $DZ=(1-S11)(1-S22)-S12S21$

We know $S12S21$ from above.

- **$1-S11=1-(-0.106065+j0.106065)=1.106065-j0.106065$**
- **$1-S22=1-(0.0855-j0.234925)=0.9145+j0.234925$**
- **$(1-S11)(1-S22)=(1.106065-j0.106065)*(0.9145+j0.234925)$**
  $=(1.106065)(0.9145)+(1.106065)(j0.234925)+(-j0.106065)(0.9145)+(-j0.106065)(j0.234925)$
  $=1.0119+j0.2598-j0.0969+0.0249$
  $=1.0368+j0.1629$

- **$DZ=(1.0368+j0.1629)-(0.086936+j0.02331)$**
  $=(1.0368-0.086936)+j(0.1629-0.02331)$
  $=0.949864+j0.13959$

Step 4: Plug values into each Z-parameter formula.

This involves more complex number multiplications, additions, and divisions. For instance, for Z11:

$Z11=Z0*((1+S11)(1-S22)+S12S21)/DZ$

- **$1+S11=1+(-0.106065+j0.106065)=0.893935+j0.106065$**
- **$(1+S11)(1-S22)=(0.893935+j0.106065)*(0.9145+j0.234925)$**
  $=0.8174+j0.2099+j0.0969-0.0249=0.7925+j0.3068$
- **Numerator of Z11 (before multiplying by Z0):**
  $(0.7925+j0.3068)+(0.086936+j0.02331)$
  $=0.879436+j0.33011$
- **Finally for Z11:**
  $Z11=50*(0.879436+j0.33011)/(0.949864+j0.13959)$
  This last step is a complex number division. You would typically convert numerator and denominator to polar form, divide magnitudes, and subtract phases, then convert back to rectangular if desired.
  $(0.879436+j0.33011)$ in polar: $0.9400\angle 20.6^\circ$
  $(0.949864+j0.13959)$ in polar: $0.9600\angle 8.35^\circ$
  Ratio: $(0.9400/0.9600)\angle(20.6^\circ-8.35^\circ)=0.979\angle 12.25^\circ$
  $Z11=50*(0.979\angle 12.25^\circ)=48.95\angle 12.25^\circ$ Ohms
  In rectangular:
  $48.95*(\cos 12.25^\circ+j\sin 12.25^\circ)=48.95*(0.977+j0.212)=47.82+j10.38$ Ohms

As you can see, this process is numerically intensive and prone to error if done manually. This is why RF engineers rely heavily on simulation software for such conversions and analyses. The key takeaway is that the relationships exist, and the transformation is based purely on the complex S-parameters and the characteristic impedance.

4.4 Analysis of RF Circuits using S-parameters

S-parameters are the native language of RF circuit design. They allow us to directly evaluate crucial performance metrics, taking into account reflections and interactions between components.

Two-Port Network Analysis:

Beyond just looking at individual S-parameter values, we often need to calculate the actual input reflection coefficient, output reflection coefficient,

and the overall gain of a two-port network when it's connected to specific source and load impedances. These are essential for designing matching networks and predicting real-world performance.

1. **Input Reflection Coefficient (Γin):**
   This parameter tells us what reflection coefficient an external source "sees" when looking into the input port (Port 1) of our two-port network, given that a specific load is connected to the output port (Port 2). This is critical for designing the source matching network.
   The formula for Γin is:
   $\Gamma in = S11 + (S12 * S21 * \Gamma L)/(1 - S22 * \Gamma L)$
   Where:
   - **S11,S12,S21,S22 are the S-parameters of the two-port network (complex numbers).**
   - **ΓL is the load reflection coefficient (a complex number) connected to Port 2. It is calculated as $\Gamma L = (ZL - Z0)/(ZL + Z0)$, where ZL is the actual load impedance and Z0 is the system characteristic impedance (e.g., 50 Ohms).**
2. **Special Case: If the load is perfectly matched to the system impedance ($ZL = Z0$), then $\Gamma L = 0$. In this case, the formula simplifies to:**
   $\Gamma in = S11$
   This confirms that S11 indeed represents the input match when the output is ideally terminated.
3. **Output Reflection Coefficient (Γout):**
   Symmetrically, Γout tells us what reflection coefficient an external load "sees" when looking into the output port (Port 2) of our two-port network, given that a specific source is connected to the input port (Port 1). This is vital for designing the load matching network.
   The formula for Γout is:
   $\Gamma out = S22 + (S12 * S21 * \Gamma S)/(1 - S11 * \Gamma S)$
   Where:
   - **S11,S12,S21,S22 are the S-parameters of the two-port network.**
   - **ΓS is the source reflection coefficient (a complex number) connected to Port 1. It is calculated as $\Gamma S = (ZS - Z0)/(ZS + Z0)$, where ZS is the actual source impedance and Z0 is the system characteristic impedance.**
4. **Special Case: If the source is perfectly matched ($ZS = Z0$), then $\Gamma S = 0$. In this case, the formula simplifies to:**
   $\Gamma out = S22$
   This confirms that S22 represents the output match when the input is ideally terminated.
5. **Transducer Power Gain (GT):**
   This is one of the most important gain definitions in RF, especially for amplifiers. It represents the ratio of the actual average power delivered

to the load (PL) to the maximum available power from the source (Pavail,S). It takes into account mismatches at both the input and output, which significantly impact real-world power transfer.
The general formula for Transducer Power Gain is:
$GT=PL/Pavail,S=(|S21|2*(1-|\Gamma S|2)*(1-|\Gamma L|2))/(|(1-S11*\Gamma S)*(1-S22*\Gamma L)-S12*S21*\Gamma S*\Gamma L|2)$
This formula looks daunting, but it accounts for all reflections. Let's look at important special cases:

- Unilateral Transducer Power Gain (GTU): This is applicable if the device is unilateral, meaning S12=0. This significantly simplifies the formula because the term $S12*S21*\Gamma S*\Gamma L$ becomes zero.
  $GTU=(|S21|2*(1-|\Gamma S|2)*(1-|\Gamma L|2))/(|1-S11*\Gamma S|2*|1-S22*\Gamma L|2)$
  This form clearly shows that for a unilateral device, the overall gain is a product of three factors:
  1. Gain from the device itself ($|S21|2$).
  2. Input matching factor ($(1-|\Gamma S|2)/|1-S11*\Gamma S|2$).
  3. Output matching factor ($(1-|\Gamma L|2)/|1-S22*\Gamma L|2$).
     Each factor accounts for the power lost due to mismatch at its respective port.
- Maximum Available Gain (GMAG): This is the maximum gain that can be achieved from an amplifier when both the input and output are simultaneously conjugately matched for maximum power transfer, and the device is unconditionally stable ($K > 1$, $|\Delta|<1$).
  $GMAG=|S21/S12|*(K-K2-1)$
  This value is a theoretical maximum and provides a benchmark for amplifier performance.

Cascaded Networks:

Many RF systems are built by connecting multiple two-port networks in series. For example, a receiver chain might consist of an LNA, followed by a filter, then a mixer, and so on. Analyzing the overall performance of such a cascaded system using individual S-parameters is a common task.

While it is possible to derive formulas for cascading S-parameters manually, they become very complex quickly. The most practical approach for cascading two networks (let's call them Network A and Network B) is:

1. Convert S-parameters to ABCD parameters for each network. (ABCD parameters are generally easier for cascading in series.)
   - $[ABCD]Total=[ABCD]A*[ABCD]B$
2. Multiply the individual ABCD matrices together to obtain the overall ABCD matrix of the cascaded system.
3. Convert the resulting total ABCD matrix back to S-parameters.

This process is almost exclusively handled by RF simulation software (e.g., Keysight ADS, Genesys, AWR Microwave Office). These tools automatically perform these complex matrix operations, allowing designers to easily simulate the overall gain, matching, and stability of an entire RF front-end by simply connecting individual component models.

Conceptual Illustration:

Imagine an LNA (Network A) connected to an RF Filter (Network B).

- LNA has S-parameters: $S_{11A}, S_{12A}, S_{21A}, S_{22A}$
- Filter has S-parameters: $S_{11B}, S_{12B}, S_{21B}, S_{22B}$
  When these are cascaded, the overall system's S-parameters ($S_{11Total}, S_{21Total}$, etc.) will depend not just on the individual gains, but also on how well S_{22A} (output match of LNA) matches S_{11B} (input match of Filter). Any mismatch between these intermediate stages will cause reflections, leading to gain ripple or overall lower gain than simply multiplying the individual S21 values. The beauty of S-parameters is that they inherently capture these interaction effects.

Numerical Example 4.4.2: Calculating Input Reflection Coefficient with a Mismatched Load (Detailed)

We will continue with the amplifier S-parameters from Example 4.2.1 at 1.8 GHz, with $Z_0=50$ Ohms:

$S_{11}=0.15\angle 135^\circ$

$S_{12}=0.02\angle -15^\circ$

$S_{21}=4.5\angle 30^\circ$

$S_{22}=0.25\angle -70^\circ$

Now, let's say this amplifier is connected to a slightly mismatched antenna with an impedance $Z_L=75-j20$ Ohms. We want to find the input reflection coefficient (Γ_{in}) seen by the previous stage (the source).

Step 1: Calculate the load reflection coefficient (Γ_L) from Z_L.

Formula: $\Gamma_L=(Z_L-Z_0)/(Z_L+Z_0)$

Given $Z_L=75-j20$ Ohms and $Z_0=50$ Ohms.

Numerator: $Z_L-Z_0=(75-j20)-50=25-j20$ Ohms

Denominator: $Z_L+Z_0=(75-j20)+50=125-j20$ Ohms

Now, perform the complex division: $\Gamma L=(25-j20)/(125-j20)$

Convert numerator and denominator to polar form:

- **Numerator: Magnitude = $252+(-20)2=625+400=1025\approx32.016$**
  Phase = $\arctan(-20/25)=\arctan(-0.8)\approx-38.66\circ$
  So, $25-j20=32.016\angle-38.66\circ$
- **Denominator: Magnitude = $1252+(-20)2=15625+400=16025\approx126.59$**
  Phase = $\arctan(-20/125)=\arctan(-0.16)\approx-9.09\circ$
  So, $125-j20=126.59\angle-9.09\circ$

Now, divide the polar forms:

$\Gamma L=(32.016/126.59)\angle(-38.66\circ-(-9.09\circ))$

$\Gamma L=0.2529\angle(-38.66\circ+9.09\circ)$

$\Gamma L=0.2529\angle-29.57\circ$

Step 2: Convert S-parameters and ΓL to rectangular form for calculations in the Γin formula.

We already have the rectangular forms for S-parameters from Example 4.3.1.

$\Gamma L=0.2529*(\cos(-29.57\circ)+j\sin(-29.57\circ))$

$\Gamma L=0.2529*(0.8697-j0.4936)=0.2199-j0.1248$

Step 3: Calculate the terms in the Γin formula:
$\Gamma in=S11+(S12*S21*\Gamma L)/(1-S22*\Gamma L)$

- **Calculate $S12*S21$ (from Example 4.3.1 - rectangular form of $0.05\angle15\circ$):**
  $S12*S21=(0.02\angle-15\circ)*(4.5\angle30\circ)=0.09\angle15\circ$
  Rectangular:
  $0.09*(\cos15\circ+j\sin15\circ)=0.09*(0.9659+j0.2588)=0.08693+j0.02329$
- **Calculate $S12*S21*\Gamma L$ (numerator of the second term):**
  $(0.09\angle15\circ)*(0.2529\angle-29.57\circ)$
  $=(0.09*0.2529)\angle(15\circ-29.57\circ)$
  $=0.02276\angle-14.57\circ$
  Rectangular: $0.02276*(\cos(-14.57\circ)+j\sin(-14.57\circ))$
  $=0.02276*(0.9676-j0.2515)=0.02203-j0.00572$
- **Calculate $1-S22*\Gamma L$ (denominator of the second term):**
  First, $S22*\Gamma L=(0.25\angle-70\circ)*(0.2529\angle-29.57\circ)$
  $=(0.25*0.2529)\angle(-70\circ-29.57\circ)$
  $=0.063225\angle-99.57\circ$
  Rectangular: $0.063225*(\cos(-99.57\circ)+j\sin(-99.57\circ))$
  $=0.063225*(-0.1663-j0.9861)$

=−0.01051−j0.06232
Now, 1−(−0.01051−j0.06232)=1+0.01051+j0.06232=1.01051+j0.06232

- **Perform the division for the second term:**
  Numerator: 0.02203−j0.00572
  Denominator: 1.01051+j0.06232
  Convert to polar:
  Num: 0.022032+(−0.00572)2=0.000485+0.0000327=0.0005177≈0.02275
  Phase: arctan(−0.00572/0.02203)=arctan(−0.2596)≈−14.56◦
  So, 0.02275∠−14.56◦
  Denom: 1.010512+0.062322=1.0211+0.00388=1.02498≈1.0124
  Phase: arctan(0.06232/1.01051)=arctan(0.06167)≈3.53◦
  So, 1.0124∠3.53◦
  Division: (0.02275/1.0124)∠(−14.56◦−3.53◦)
  =0.02247∠−18.09◦
  Rectangular: 0.02247∗(cos(−18.09◦)+jsin(−18.09◦))
  =0.02247∗(0.9507−j0.3105)=0.02136−j0.00698

Step 4: Add S11 to the result of the second term.

S11=−0.106065+j0.106065

Γin=(−0.106065+j0.106065)+(0.02136−j0.00698)

Γin=(−0.106065+0.02136)+j(0.106065−0.00698)

Γin=−0.084705+j0.099085

Step 5: Convert final Γin to polar form (often preferred for reflection coefficients).

Magnitude:
∣Γin∣=(−0.084705)2+(0.099085)2=0.007175+0.009818=0.016993≈0.13036

Phase: arctan(0.099085/−0.084705)=arctan(−1.1697)≈−49.49◦. Since the real part is negative and imaginary is positive, this angle is in the second quadrant. So, 180◦−49.49◦=130.51◦.

So, Γin=0.13036∠130.51◦

Interpretation: With the mismatched load (ZL=75−j20 Ohms), the input reflection coefficient is 0.13036∠130.51◦.

Compare this to S11=0.15∠135◦. The magnitude has slightly decreased, meaning the input match has slightly improved compared to the ideal 50 Ohm load case. This shows how the load termination can affect the input impedance of a bilateral device, which is a critical consideration for RF system

design. This kind of detailed calculation is often done by RF simulation software.

4.5 Stability Analysis

Stability is arguably the most critical aspect of RF amplifier design. An amplifier is stable if it remains free from unwanted oscillations under all specified operating conditions. An unstable amplifier will not perform its intended function; instead, it will act as an oscillator, converting DC power into unwanted RF signals, potentially damaging components, or severely degrading system performance. S-parameters provide direct methods to assess the stability of active two-port networks.

Unilateral vs. Bilateral Networks:

The concept of unilateral or bilateral nature is fundamental to understanding feedback and stability.

- Unilateral Network: An ideal unilateral network is a theoretical construct where there is absolutely no signal transmission or feedback from the output port back to the input port.
    - S-parameter Condition: For a two-port network, this means $S12=0$.
    - Implication: If a device is truly unilateral, its input characteristics (like Γin) are completely independent of the load connected to its output, and its output characteristics (like Γout) are completely independent of the source connected to its input. This significantly simplifies design, as input and output matching networks can be designed independently.
    - Reality: Perfect unilateralism is rarely achieved in real active devices like transistors due to unavoidable parasitic capacitances and inductances that provide a feedback path. However, many RF amplifiers are designed to be "approximately unilateral" by ensuring very high reverse isolation (very small $|S12|$).
- Bilateral Network: A bilateral network is one where there is some degree of signal transmission or feedback from the output back to the input, meaning $S12=0$.
    - Reality: Almost all practical active devices at RF frequencies are bilateral. Even a tiny S12 can become significant at high frequencies or high gain.
    - Implication: The input impedance of a bilateral device is dependent on the load connected to its output (Γin depends on ΓL), and its output impedance is dependent on the source connected to its input (Γout depends on ΓS). This interdependence makes the design of simultaneous matching networks and the analysis of stability more complex. If the internal feedback (S12) combined with external source and load

reflections creates a loop gain greater than unity with a phase shift of 360 degrees (or 0 degrees), the device will oscillate.

Conditions for Unconditional Stability:

An active two-port network (like a transistor or an amplifier stage) is considered unconditionally stable if it will remain stable (i.e., not oscillate) regardless of what passive source impedance (ZS, corresponding to $|\Gamma S| \le 1$) or passive load impedance (ZL, corresponding to $|\Gamma L| \le 1$) is connected to it. This is the most desirable characteristic for a general-purpose amplifier that needs to operate reliably in various system environments.

The unconditional stability of a two-port network can be mathematically determined from its S-parameters using two key criteria: the K-factor (Rollett stability factor) and the Delta (Δ) parameter.

The conditions for unconditional stability are:

1. **K > 1: The K-factor (stability factor) must be greater than 1.**
   $K=(1-|S11|2-|S22|2+|\Delta|2)/(2*|S12*S21|)$
   Where Δ (Delta) is the determinant of the S-matrix, calculated as:
   $\Delta=S11*S22-S12*S21$
2. **$|\Delta|<1$: The magnitude of the determinant of the S-matrix must be less than 1.**

Interpretation of Stability Conditions:

- **If K > 1 AND $|\Delta|<1$: The network is unconditionally stable. This is the ideal scenario for an amplifier. You can connect any passive source and load, and the amplifier will not oscillate. This provides great flexibility in design.**
- **If K < 1: The network is conditionally stable. This means the device can be made stable for certain source and load terminations, but there exist specific passive source and load impedances that will cause it to oscillate. In this case, designers must use stability circles (a graphical tool plotted on the Smith Chart, which will be covered in a later module) to identify the regions of source and load impedances that cause instability. The matching networks must then be designed to avoid these regions. This requires more careful design and analysis.**
- **If K = 1: The network is marginally stable, sitting right at the boundary between unconditional and conditional stability. Any slight change in parameters or operating conditions could push it into instability.**

Physical Meaning of K and Δ:

- The K-factor essentially quantifies the inherent "stability margin" of the device. It compares the internal positive feedback (related to $S12*S21$) to the reflections at the input and output. A higher K-factor implies that the device is less likely to oscillate.
- The Δ parameter (determinant of the S-matrix) is also related to the internal feedback and transfer characteristics of the device. The condition $|\Delta|<1$ is necessary to ensure that the network is "passive at the boundary," meaning it cannot self-oscillate simply from energy circulating within the network itself when terminated reactively.

Numerical Example 4.5.1: Stability Analysis using K and Delta (Detailed Steps)

Let's evaluate the stability of a transistor at a specific operating point and frequency, say 8 GHz, with the following measured S-parameters:

$S11=0.9\angle -120^\circ$

$S12=0.08\angle 60^\circ$

$S21=3.0\angle 90^\circ$

$S22=0.6\angle -45^\circ$

Step 1: Calculate the magnitude squared of each S-parameter.

These are used in the K-factor formula.

- $|S11|^2=(0.9)^2=0.81$
- $|S12|^2=(0.08)^2=0.0064$
- $|S21|^2=(3.0)^2=9.0$
- $|S22|^2=(0.6)^2=0.36$

Step 2: Calculate Delta ($\Delta=S11*S22-S12*S21$).

This requires converting S-parameters to rectangular form first, performing complex multiplication, and then complex subtraction.

- Convert to Rectangular Form:
  - $S11=0.9*(\cos(-120^\circ)+j\sin(-120^\circ))=0.9*(-0.5-j0.8660)=-0.45-j0.7794$
  - $S12=0.08*(\cos 60^\circ+j\sin 60^\circ)=0.08*(0.5+j0.8660)=0.04+j0.06928$
  - $S21=3.0*(\cos 90^\circ+j\sin 90^\circ)=3.0*(0+j1)=0+j3.0$
  - $S22=0.6*(\cos(-45^\circ)+j\sin(-45^\circ))=0.6*(0.7071-j0.7071)=0.42426-j0.42426$
- Calculate $S11*S22$:
  $S11*S22=(-0.45-j0.7794)*(0.42426-j0.42426)$

$=(-0.45)(0.42426)+(-0.45)(-j0.42426)+(-j0.7794)(0.42426)+(-j0.7794)(-j0.42426)$
$=-0.1909+j0.1909-j0.3308+(-1)(0.3308)$
$=(-0.1909-0.3308)+j(0.1909-0.3308)$
$=-0.5217-j0.1399$

- **Calculate $S12*S21$:**
  $S12*S21=(0.04+j0.06928)*(0+j3.0)$
  $=(0.04)(0)+(0.04)(j3.0)+(j0.06928)(0)+(j0.06928)(j3.0)$
  $=0+j0.12+0+(-1)(0.20784)$
  $=-0.20784+j0.12$
- **Calculate $\Delta=S11*S22-S12*S21$:**
  $\Delta=(-0.5217-j0.1399)-(-0.20784+j0.12)$
  $\Delta=(-0.5217+0.20784)+j(-0.1399-0.12)$
  $\Delta=-0.31386-j0.2599$
- **Calculate Magnitude of Δ ($|\Delta|$):**
  $|\Delta|=(-0.31386)2+(-0.2599)2$
  $|\Delta|=0.0985+0.0675=0.166\approx0.4074$

Step 3: Check the $|\Delta|<1$ condition.

$|\Delta|=0.4074$. Since $0.4074<1$, this condition is met.

Step 4: Calculate the K-factor.

$K=(1-|S11|2-|S22|2+|\Delta|2)/(2*|S12*S21|)$

- First, calculate the magnitude of the product $|S12*S21|$ for the denominator.
  It's simply the product of their magnitudes: $|S12|*|S21|$.
  $|S12*S21|=(0.08)*(3.0)=0.24$
- Now substitute all values into the K-factor formula:
  $K=(1-0.81-0.36+(0.4074)2)/(2*0.24)$
  $K=(1-0.81-0.36+0.166)/0.48$
  $K=(-0.004)/0.48$
  $K=-0.00833$

Step 5: Check the K > 1 condition.

$K=-0.00833$. Since K is not greater than 1 (it's actually negative), this condition is NOT met.

Conclusion of Stability Analysis:

- Condition 1: $|\Delta|<1$ is MET ($0.4074<1$).
- Condition 2: $K > 1$ is NOT MET ($-0.00833<1$).

Because K is less than 1, the transistor at this operating point and frequency is conditionally stable. This means the device will oscillate if terminated with certain passive source and load impedances. A designer would need to carefully map out the unstable regions on a Smith Chart using stability circles to ensure that the chosen source and load matching networks keep the amplifier out of oscillation. This is a critical step in RF amplifier design to guarantee proper operation.

This detailed, numerical approach allows for a precise understanding of the theoretical concepts and their practical application in RF circuit analysis and design.
